

Multi-Scenario Multi-Criteria Optimization in Engineering Design

MARGARET M. WIECEK AND VIJAY SINGH
Department of Mathematical Sciences
Clemson University
Clemson, SC 29634-0975

VINCENT BLOUIN
Department of Mechanical Engineering
Clemson University
Clemson, SC 29634-0921

Abstract

Motivated by applications in engineering design, a mathematical model of the multi-scenario multi-criteria optimization problem is introduced. Theoretical results for the single-scenario case are presented to support a solution methodology developed for the bi-scenario bi-criteria case. Multi-scenario design problems are traditionally solved by aggregation of all objectives of all scenarios into a large multi-criteria problem. The difficulties that arise from this approach are highlighted. The proposed methodology is a scenario-based approach where a design problem is solved for each scenario resulting into multiple sets of solutions. The methodology is developed as an exploration tool of these solution sets in both the design and objective spaces. The methodology is applicable to problems with large numbers of scenarios and/or criteria. Mathematical and structural examples are included to illustrate the implementation of the methodology, its strengths and weaknesses.

Keywords: *multiple scenarios, multi-criteria optimization, Pareto outcomes, efficient designs*

1. Introduction

With the advancement of technology, the design process is becoming increasingly complex. The industrial competitiveness coupled with the globalization of the economy has forced the engineering and science communities to look for designs that are good not only for a single application but several applications grouped together. Consider a process of designing a car with special attention to two criteria, cost and reliability. A typical customer would like to have a car with minimum cost and maximum reliability but a higher level of reliability usually results in a higher cost. This design problem can be then modeled as an optimization problem with two non-commensurate and conflicting criteria. Additionally, the car can be designed for various driving conditions (e.g., interstate highway, unpaved road), or for different markets (e.g., American, European, Asian), or for different types of use (e.g., family car, transportation of goods, taxi), or for some other types of scenarios. It is then of interest to design a reliable and inexpensive car performing well in some of those scenarios. In every scenario the criteria may have different mathematical representations, although their physical interpretation remains the same, that of minimizing cost and maximizing reliability. In some other applications, not only the

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mathematical representation but also the physical meaning of the criteria as well as the design space may vary from scenario to scenario. As a result, a design process under different scenarios for the same physical problem leads to the problem of multi-scenario multi-criteria optimization.

To our knowledge, the concept of multi-scenario optimization has not been introduced in the literature although some scientists and engineers studied optimization problems in this context. Below we discuss a monograph and several articles in which scenario oriented optimization problems have been formulated with a scenario being related to data instance, a product platform, or a maneuver.

Kouvelis and Yu [11] studied multi-scenario single-criterion optimization problems. They proposed the concept of robust solution that would be ‘robust’ for the same mathematical model associated with multiple data scenarios. A multi-scenario single criterion optimization problem was converted into a robust (single-scenario single-criterion) problem using the min-max formulation. The absolute robust solution was defined to minimize the maximum criterion value selected from among all realizable scenarios over all feasible designs. As a result, finding the robust design was based on multi-criteria optimization.

Traditional design process considers designing a single product. However, since the nineteen nineties product platform design has been studied. A product platform is a set of common components or parts from which several variations of a product can be made. Product platform design requires the selection of shared parts and the assessment of potential sacrifices in individual product performance resulting from parts sharing. In platform design models some design variables that are common to the products of a particular platform are kept at the same level through the commonality constraints. Fujita and Ishii [8], Fujita et al. [7], and Simpson et al. [19] associated a criterion with each product in a platform and grouping the products (criteria) solved the resulting multi-criteria problem. Nelson et al. [12] modeled a product platform as a single multi-criteria optimization problem and an optimal platform design was in the Pareto set of this problem. A collection of platforms (combinations of common parts) resulted in several multi-criteria problems and several Pareto sets to be analyzed simultaneously. An optimal platform design for a collection was defined to belong to one of these Pareto sets associated with a certain platform in the collection, which was chosen based on product performance and other factors. Fellini et al. [6] applied multi-criteria optimization-based product platform design to a family of automotive powertrains. In general, a product platform represents a particular scenario of the problem and several product platforms correspond to several scenarios. Depending on the number of products in a platform, a multi-criteria problem can be formulated for each scenario. For example, if there are two products in a platform, a bi-criteria problem can be formulated by associating some type of performance criterion to each of the products.

Vehicle design is another area in which two or more multi-criteria models can be analyzed simultaneously. Efforts have been undertaken to optimize vehicle performance indices in different operating scenarios or maneuvers. Gobbi and Mastinu [9] applied multi-criteria optimization to find a best compromise between several performance indices when the vehicle is driven on roads with changing roughness. Chakravartula [3] developed a simulation-based methodology to heavy vehicle design for eight performance indices across eight driving maneuvers.

In view of the applications discussed above we intend to formalize the research efforts in which collections of multi-criteria problems have been used. We introduce the notion of multi-

scenario multi-criteria optimization and formulate the problem mathematically. We argue that a scenario-oriented approach can become a more flexible design tool than the classical all-at-once approach in which one solves an optimization problem with all criteria neglecting their relevance to particular scenarios. We believe that the all-at-once approach eliminates the possibility of customizing the design while the scenario-oriented approach allows for a detailed analysis of each scenario individually. With optimization-based design moving these days in the direction of mass customization, one needs a tool for designing products for multiple uses performing well in multiple scenarios.

Furthermore, in the traditional all-at-once approach the following three exertions become increasingly complex with the number of criteria: (i) preference setting and decision making; (ii) physical and geometrical perception of the problem; and (iii) visual and graphical representation of the solution set, which is limited to three or four dimensions. As a result, it is clear that dealing with a series of simple problems, i.e., one per scenario, is easier than with a large all-at-once problem. Even though, as it will be shown in the following sections, all the solutions of the all-at-once approach cannot be found by the scenario-oriented approach, we believe that the advantages of the latter should be exploited. The challenge, which is the core of this research, is to study the similarities between the sets of solutions of the two methods and to show that the scenario-oriented approach is an attractive design tool.

Let $S = \{1, 2, \dots, N\}$ be the set of scenarios (platforms, maneuvers, etc.). We define *the multi-scenario multi-criteria problem* as follows

$$\begin{aligned} \min \quad & \{[f_1^s(x), f_2^s(x), \dots, f_{m(s)}^s(x)], s \in S\} \\ \text{s.t.} \quad & x \in X^1 \cap X^2 \cap \dots \cap X^N \end{aligned} \tag{1.1}$$

where every scenario $s \in S$ is modeled by $m(s)$ real-valued criterion functions, $f_j^s : \mathbb{R}^n \rightarrow \mathbb{R}^1, j = 1, \dots, m(s)$, to be minimized over a set of feasible designs $X^s \subseteq \mathbb{R}^n$, with \mathbb{R}^n being the n -dimensional Euclidean space and n the number of design variables. Problem (1.1) incorporates N multi-criteria problems, each with possibly different criterion functions minimized over a different feasible set. While solving each multi-criteria problem is understood as finding its Pareto solutions, the idea for solving problem (1.1) may not be clear. Before we explicitly introduce solution concepts for problem (1.1) in Section 3, based on motivating applications we expect problem (1.1) to have a solution good or satisfactory for all N scenarios. Ideally, that solution should be ‘optimal’ for all scenarios. The challenge now is to define the optimality, examine whether such an optimal solution exists, and if so, find it.

The paper is organized as follows. Section 2 is concerned with the single-scenario multi-criteria case. We review well-established techniques for finding and/or approximating Pareto sets of multi-criteria problems, examine the efficiency of solutions when the number of criterion functions changes, and evaluate the lack of efficiency of designs. In Section 3, we first present simple mathematical examples highlighting the issues of concern when dealing with two bi-criteria problems simultaneously. We define the optimality concept for the bi-scenario case and propose a scenario-oriented approach to finding optimal solutions in which the results of Section 2 are employed. The approach uses extensively the capability of approximating the Pareto set and representing it graphically. Section 4 includes engineering examples illustrating the methodology and Section 5 concludes the paper.

2. The Single-Scenario Case

In order to enhance our understanding of the all-at-once approach and a scenario-oriented approach, we are interested in studying the link between their respective solution sets. Consider the case of a single scenario, i.e., $N = 1$ and $m(s) = m$. Dropping the superscripts, problem (1.1) reduces to

$$\begin{aligned} \min \quad & [f_1(x), f_2(x), \dots, f_m(x)] \\ \text{s.t.} \quad & x \in X \subseteq \mathbb{R}^n \end{aligned} \tag{2.1}$$

which is the well-known multi-criteria minimization problem whose solving is understood as finding its Pareto outcomes and efficient designs.

A design $x^0 \in X$ is said to be *efficient* for problem (2.1) if there is no $x \in X$, $x^0 \neq x$, such that $f_i(x) \leq f_i(x^0)$, $i = 1, \dots, m$, with a strict inequality for at least one index i . The set of all efficient designs of problem (2.1) is denoted by X_E . The set of all feasible criterion vectors of problem (2.1) defined as $Z = \{z \in \mathbb{R}^m : z = f(x), x \in X\} = f(X)$, where $f(x) = [f_1(x), \dots, f_m(x)]$ is referred to as the *set of outcomes* in the objective space. The image $z^0 = f(x^0)$ of an efficient design x^0 is called a *Pareto outcome* of problem (2.1), and the set of all Pareto outcomes is denoted by Z_E .

Pareto sets typically include infinitely many points and therefore it is usually of interest to find a preferred efficient design (Pareto outcome) based on designer's preferences additionally introduced to the problem.

2.1. Solving multi-criteria optimization problems

The essence of multi-criteria optimization is to find or approximate the Pareto set in the objective space and its pre-image, the efficient set in the design space. Significant results have been accomplished on this subject. Early efforts focused on developing methods for generating selected points of these sets (the weighted-sum method [10], the ϵ -constraint method [2, 4], and the weighted-Tchebycheff method [5, 20, 21, 23], and others). Most recently, vast progress has been made towards developing methodologies to approximate the Pareto set [16]. With highly developed computer power, graphical representations of the Pareto set have become a fundamental tool for evaluating Pareto solutions [13].

In this paper, solving a multi-criteria optimization problem is understood as finding numerical point-wise representations of the Pareto and efficient sets. While any approximation approach from the literature could be used, we produce point-wise approximations with the weighted-Tchebycheff method and the norm-based method [17, 18]. Graphically, both methods yield a piecewise linear approximation of the Pareto set with all extreme points being Pareto. Numerically, they produce a set of Pareto points and their pre-images, the efficient points. The methods are suitable for a broad class (continuous and/or discrete, convex and/or nonconvex) of bi-criteria problems.

2.2. Efficient solutions for problems with a variable number of criteria

In this section we study the conditions under which efficiency of a design is preserved while going from an m to an $(m+p)$ -dimensional criterion space, where $p \geq 1$. We also consider the

consequences of reverting back from an $(m+p)$ to an m -dimensional space. We will use these results in Section 3.

From m to $(m+p)$ -dimensional criteria space

Example 2.2.1 Consider the bi-criteria problem with three feasible designs

$$\begin{aligned} \min \quad & [f_1(x), f_2(x)] \\ \text{s.t.} \quad & x \in X = \{x^1, x^2, x^3\} \end{aligned}$$

Let $f(x^1) = [1, 2]$, $f(x^2) = [2, 1]$, and $f(x^3) = [3, 4]$. Then x^1 and x^2 are both efficient for this problem. Assume now that this problem has been modified by adding one criterion yielding the three-criteria problem

$$\begin{aligned} \min \quad & [f_1(x), f_2(x), f_3(x)] \\ \text{s.t.} \quad & x \in X = \{x^1, x^2, x^3\} \end{aligned}$$

Let $f(x^1) = [1, 2, 3]$, $f(x^2) = [2, 1, 7]$, and $f(x^3) = [3, 4, 1]$, which makes x^1 , x^2 as well as x^3 efficient for the new problem. We therefore find that the bi-criteria problem cannot capture all the efficient points of the three-criteria problem.

Example 2.2.2 Consider the bi-criteria problem

$$\begin{aligned} \min \quad & [f_1(x), f_2(x)] \\ \text{s.t.} \quad & x \in X = \{x^1, x^2\}, \end{aligned}$$

where $x^1 \neq x^2$. Let $f(x^1) = [1, 2]$ and $f(x^2) = [1, 2]$, which makes both x^1 and x^2 efficient for this problem. Assume now that this problem has been modified by adding one additional criterion yielding the three-criteria problem

$$\begin{aligned} \min \quad & [f_1(x), f_2(x), f_3(x)] \\ \text{s.t.} \quad & x \in X = \{x^1, x^2\}. \end{aligned}$$

Now let $f(x^1) = [1, 2, 3]$ and $f(x^2) = [1, 2, 4]$, and x^1 becomes the only efficient point for this new problem. We therefore observe that one of the efficient points of the bi-criteria problem is no longer efficient for the three-criteria problem.

In order to formalize these observations we need the definition of injective mapping. Consider a criterion f_i , $i = 1, \dots, m$, as given in problem (2.1), which is a real-valued mapping from X to Z_i , where $Z_i = \{z \in \mathbb{R}^1 : z = f_i(x), x \in X\}$. A criterion f_i , $i = 1, \dots, m$, of problem (2.1) is called *injective* (or one-one) if whenever $x^1 \neq x^2$ then $f_i(x^1) \neq f_i(x^2)$, or equivalently, $f_i(x^1) = f_i(x^2)$ then $x^1 = x^2$, where $x^1, x^2 \in X$.

Theorem 2.2.1 Consider the following two multi-criteria problems of form (2.1) composed of m and $m+p$ criterion functions, respectively, where $p \geq 1$:

$$\begin{aligned} \min \quad & [f_1(x), \dots, f_m(x)] \\ \text{s.t.} \quad & x \in X \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} \min \quad & [f_1(x), \dots, f_m(x), f_{m+1}(x), \dots, f_{m+p}(x)] \\ \text{s.t.} \quad & x \in X \end{aligned} \quad (2.3)$$

If there exists $i \in \{1, \dots, m\}$ such that f_i is injective, then any efficient solution of problem (2.2) is efficient for problem (2.3).

Proof Let X_{E1} and X_{E2} denote the efficient sets of problems (2.2) and (2.3), respectively. Let $x \in X_{E1} \Leftrightarrow$ there is no $x^* \in X$, $x^* \neq x$ such that $f_i(x^*) \leq f_i(x)$, $i = 1, \dots, m$, and $f_i(x^*) < f_i(x)$ for at least one i , $i \in \{1, \dots, m\}$. By contradiction, suppose $x \notin X_{E2} \Leftrightarrow$ there exists an $x' \in X$, $x' \neq x$, such that

$$f_i(x') \leq f_i(x) \text{ for } i = 1, \dots, m+p \quad (2.4)$$

and $f_k(x') < f_k(x)$ for at least one k , $k \in \{1, \dots, m+p\}$.

Case 1 Suppose that for $k \in \{m+1, \dots, m+p\}$ we have $f_k(x') < f_k(x)$. From (2.4)

$$f_i(x') \leq f_i(x) \text{ for } i = 1, \dots, m \quad (2.5)$$

Since $x' \neq x$ and f_j is injective for a $j \in \{1, \dots, m\}$, then

$$f_j(x') \neq f_j(x) \quad (2.6)$$

Expressions (2.5) and (2.6) imply that $f_i(x') \leq f_i(x)$, $i = 1, \dots, m$, and $f_j(x') < f_j(x)$ for a $j \in \{1, \dots, m\}$, which contradicts the assumption that $x \in X_{E1}$. Hence $x \in X_{E2}$.

Case 2 Suppose that for $k \in \{1, \dots, m\}$ we have $f_k(x') < f_k(x)$. This together with (2.4) immediately contradicts the assumption that $x \in X_{E1}$. Hence $x \in X_{E2}$.

Case 3 Suppose $f_i(x') < f_i(x)$ for $i = 1, \dots, m+p$. This immediately contradicts the assumption that $x \in X_{E1}$. Hence $x \in X_{E2}$.

Remark 2.2.1 The condition that f_i is injective for at least one index i in problem (2.2) is needed for Case 1 of the proof above in which the strict inequality holds for a criterion k present in problem (2.3) but not in problem (2.2). If the strict inequality holds for one of the m criteria of problem (2.2), then this condition is not needed.

Remark 2.2.2 If none of the criterion functions of problem (2.2) is injective, cases 2 and 3 of the proof above still hold. However in case 1, $f_i(x')$ can be equal to $f_i(x)$, $i = 1, \dots, m$, even when $x' \neq x$. This simply means that two different designs produce the same Pareto outcome in the criteria space and the theorem does not hold.

From $(m+p)$ to m -dimensional criteria space

We now examine efficiency of solutions when one moves from an $(m+p)$ -dimensional criteria space to an m -dimensional criteria space. Again, we first present an example.

Example 2.2.3 Consider the three-criteria problem

$$\begin{aligned} \min \quad & [f_1(x), f_2(x), f_3(x)] \\ \text{s.t.} \quad & x \in X = \{x^1, x^2\} \end{aligned}$$

(i) Let $f(x^1) = [1, 1, 2]$ and $f(x^2) = [2, 3, 1]$. Then x^1 and x^2 are both efficient for this problem. Now suppose that the criterion f_3 is dropped which results in the bi-criteria problem. In this case $f(x^1) = [1, 1]$, $f(x^2) = [2, 3]$ and x^1 becomes the only efficient point for the new problem.

(ii) Let $f(x^1) = [3, 2, 6]$ and $f(x^2) = [4, 1, 5]$. In this case x^1 and x^2 are both efficient for the three-criteria problem. If the criterion f_3 is dropped, $f(x^1) = [3, 2]$, $f(x^2) = [4, 1]$ and x^1 and x^2 remain efficient for the bi-criteria problem.

Remark 2.2.4 Given two efficient points, if the conflict between criterion functions is only in one criterion, one of these points is not efficient when that criterion has been dropped.

Remark 2.2.5 Given two efficient points, if the conflict between the criterion functions is within a certain group of criteria, both points remain efficient when other criteria not being in this group have been dropped.

Engineering relevance

The analysis above shows that changing the dimension of the objective space may significantly affect the structure of the efficient set. It is also obviously difficult in practice to check the injectivity condition and, in fact, one may expect that this condition is not satisfied for many engineering applications. Despite these facts, we still advocate solving a series of smaller multi-criteria problems rather than the all-at-once problem due to already presented arguments and other features illustrated later.

2.3. Quantification of lack of efficiency

The scenario-oriented approach proposed in this paper makes use of quantification of the lack of efficiency of a design. For a multi-criteria optimization problem, consider a Pareto set and a design $x^0 \in X$ that is not efficient, i.e., $x^0 \notin X_E$. The lack of efficiency of x^0 , denoted by $LOE(x^0)$, can be quantified with two approaches: (i) Benson's method [2] and (ii) the approximation-based method proposed in this paper.

Benson's method

Benson [2] developed a method to either inform the designer that a design x^0 is efficient for a multi-criteria optimization problem or quantify the design's lack of efficiency for the problem. To apply Benson's method for the bi-criteria case (let $m = 2$ in (2.1)), the following single-criterion maximization problem must be solved.

$$\begin{aligned}
 \max \quad & w(t_1, t_2) = t_1 + t_2 \\
 \text{w.r.t} \quad & x, t_1, t_2 \\
 \text{s.t.} \quad & f_1(x) + t_1 - f_1(x^0) = 0 \\
 & f_2(x) + t_2 - f_2(x^0) = 0 \\
 & t_1 \geq 0, \quad t_2 \geq 0 \\
 & x \in X
 \end{aligned} \tag{2.7}$$

Let (x^B, t_1^B, t_2^B) be an optimal solution of problem (2.7) with the optimal objective value $w^B = w(t_1^B, t_2^B)$. If $w^B = 0$ then the candidate design x^0 is efficient for the problem (and also

$x^B = x^0$). If $w^B > 0$ then the candidate design x^0 is not efficient for the problem and the design x^B produced by (2.7) is efficient. From the equality constraints of (2.7), the quantity w^B can be written as

$$w^B = t_1^B + t_2^B = f_1(x^0) - f_1(x^B) + f_2(x^0) - f_2(x^B)$$

and interpreted as the difference between the outcomes of candidate design x^0 and efficient design x^B . This positive quantity w^B measures $\text{LOE}(x^0)$ with respect to the problem. The concept is illustrated in Figure 1.

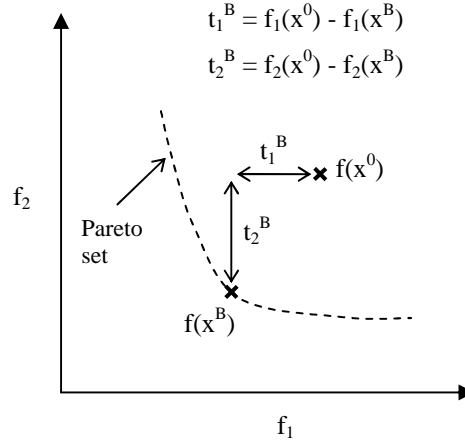


Figure 1. Quantification of lack of efficiency of design x^0 by Benson's method [2]

Approximation-based approach

As an alternative to Benson's method for the quantification of lack of efficiency of a design, we introduce the approximation-based approach. While Benson's method was developed under the assumption that the Pareto set remains unknown to the designer, we make use of the fact that an approximation of the Pareto set has been constructed and is available. We propose to quantify the lack of efficiency of a design by measuring a distance in the objective space between $f(x^0)$, the outcome of x^0 , and the available approximation:

$$\text{LOE}(x^0) = \min_{z \in \text{APR}(Z_E)} |f(x^0), z|$$

where $\text{APR}(Z_E)$ denotes the approximation of the Pareto set Z_E , and $|\cdot, \cdot|$ denotes a distance measure derived from a norm of choice. For bi-criteria problems the approximating set comes in the form of a piecewise linear curve when a suitable approximation approach is used (see Section 2.1).

As illustrated in Figure 2, $\text{LOE}(x^0)$ may be calculated as the minimum Euclidean distance between $f(x^0)$ and the approximating curve. While the issue of the choice of the norm measuring the distance will be addressed in the future, the Euclidean norm is used in the experiments presented in this paper.

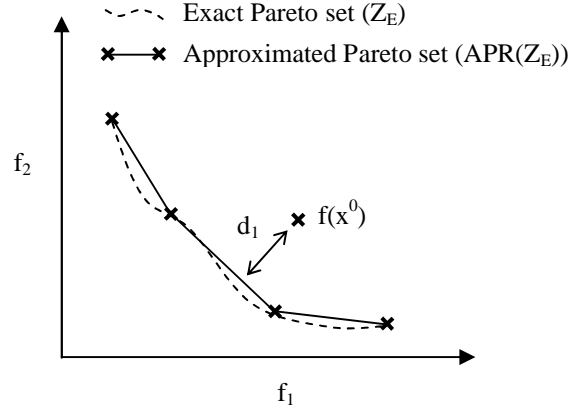


Figure 2. Quantification of lack of efficiency of design x^0 by the approximation-based approach based on Euclidean distance

3. The Bi-Scenario Case

Problem (1.1) yields a multi-criteria minimization problem of form (2.1) for each $s \in S$. All the concepts presented in section 2 can be defined in an analogous manner for each of the problems parameterized by the scenario s .

Consider *the bi-scenario bi-criteria problem* being a specific case of problem (1.1) with $S = \{1, 2\}$, $m(1) = m(2) = 2$:

$$\begin{aligned} \min \quad & \{[f_1^s(x), f_2^s(x)], s = 1, 2\} \\ \text{s.t.} \quad & x \in X^1 \cap X^2 \end{aligned} \quad (3.1)$$

Problem (3.1) is composed of two bi-criteria problems, namely the common design space problem 1 (CDSP1) for $s = 1$ and the common design space problem 2 (CDSP2) for $s = 2$:

$$\begin{aligned} \min \quad & [f_1^s(x), f_2^s(x)] = f^s(x) \\ \text{s.t.} \quad & x \in X^1 \cap X^2 \end{aligned} \quad (3.2)$$

Let X_{EC}^s and Z_{EC}^s , $s = 1, 2$, denote the efficient sets and the Pareto sets of CDSPs 1 and 2, respectively. Problem (3.1) can also be partitioned into the individual design space problem 1 (IDSP1):

$$\begin{aligned} \min \quad & [f_1^1(x), f_2^1(x)] \\ \text{s.t.} \quad & x \in X^1 \end{aligned} \quad (3.3)$$

and the individual design space problem 2 (IDSP2):

$$\begin{aligned} \min \quad & [f_1^2(x), f_2^2(x)] \\ \text{s.t.} \quad & x \in X^2 \end{aligned} \quad (3.4)$$

Let X_{EI}^s and Z_{EI}^s , $s = 1, 2$, denote the efficient sets and the Pareto sets of IDSPs 1 and 2, respectively.

We call attention to the difference between ‘common designs’ and ‘common efficient designs’. It is clear that a multi-scenario optimization problem is meaningless if there is no

common design between scenarios due to empty intersection between design spaces X^s , $s \in S$. Common efficient designs are those that are common and efficient for both scenarios. Therefore these designs are generally of prime interest to the designers and decision makers. However, as explained later, these common efficient designs may not be preferred over some other satisfactory solutions. We obtain additional insight into these problems analyzing simple mathematical examples that now follow.

3.1. Examples

In this section, working with example problems, we review the information from analyzing Pareto sets and their pre-image in the design space. In an attempt to facilitate the understanding of this paper and since several concepts may appear confusing to the novice reader, we discuss typical errors that may lead to premature and erroneous conclusions.

Example 3.1.1 Consider the following bi-scenario bi-criteria problem in which $X^1 = X^2 = X$:

$$\begin{aligned} \min \quad & \left\{ [(x_1 - 2)^2 + (x_2 - 1)^2, x_1^2 + (x_2 - 3)^2], [(x_1 - 3)^2 + (x_2 - 4)^2, (x_1 - 4)^2 + x_2^2] \right\} \\ \text{s.t.} \quad & x \in X \end{aligned}$$

where $X = \{(x_1, x_2) \in \mathbb{R}^2 : x_2^2 - 0.5x_1 \leq 0, (x_1 - 2)^2 + x_2^2 - 4 \leq 0, x_1 \geq 0, x_2 \geq 0\}$. Figure 3 depicts the design space of this problem as well as the efficient sets.

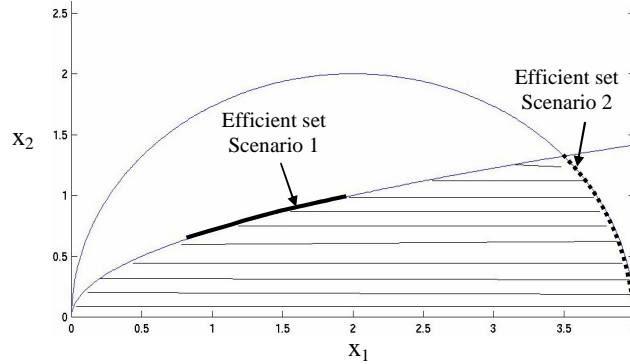


Figure 3. Design space and efficient sets of Example 3.1.1

Since the IDSPs and the CDSPs have the same design space, they are equivalent. Figure 4 shows the plots of the Pareto points of these scenarios. Whether a common efficient design exists or not can be examined by checking the efficient sets in the design space (Figure 3) based on the knowledge of the Pareto sets. We find that the two scenarios have no efficient design in common.

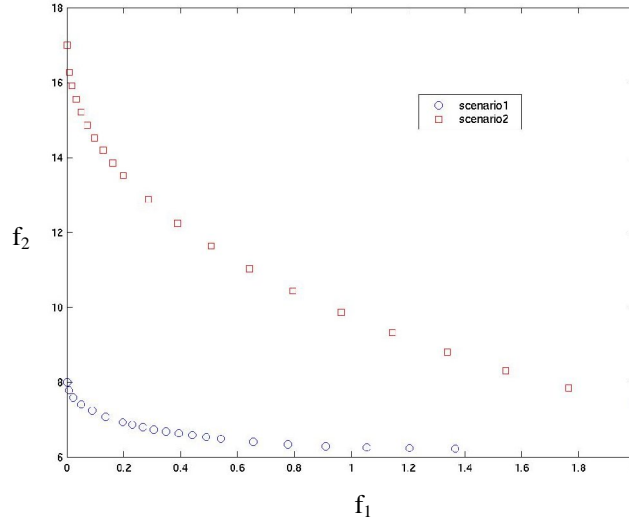


Figure 4. Pareto sets for the IDSPs and CDSPs of Example 3.1.1

Example 3.1.2 In this example the design space of the previous example is retained and again is common for two scenario problems: $X^1 = X^2 = X$. Consider the following problem:

$$\begin{aligned} \min \quad & \left\{ [(x_1 - 4)^2 + (x_2 - 1)^2, x_1^2 + (x_2 - 4)^2], [(x_1 - 5)^2 + (x_2 - 4)^2, (x_1 - 6)^2 + (x_2 - 2)^2] \right\} \\ \text{s.t.} \quad & x \in X \end{aligned}$$

Once again, the IDSPs and the CDSPs are equivalent. Figure 5 shows the design space and the efficient sets interpolated from the collection of 21 computed points depicted in Table 1. Figure 6 shows the Pareto sets of these problems.

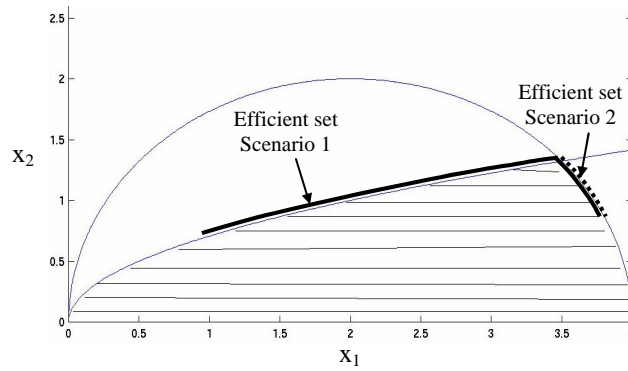


Figure 5. Design space and efficient sets of Example 3.1.2

Table 1. Efficient set for the IDSPs and CDSPs of Example 3.1.2

Scenario 1		Scenario 2	
x_1	x_2	x_1	x_2
1.099	0.741	3.500	1.323
1.278	0.799	3.521	1.298
1.461	0.855	3.542	1.274
1.648	0.908	3.562	1.248
1.836	0.958	3.582	1.223
2.026	1.006	3.602	1.197
2.121	1.030	3.621	1.171
2.217	1.053	3.640	1.145
2.313	1.075	3.658	1.118
2.409	1.097	3.676	1.091
2.505	1.119	3.694	1.064
2.602	1.141	3.702	1.050
2.787	1.180	3.711	1.036
2.880	1.200	3.719	1.022
2.973	1.219	3.727	1.008
3.159	1.257	3.735	0.994
3.345	1.293	3.743	0.980
3.500	1.323	3.751	0.966
3.616	1.178	3.759	0.952
3.708	1.040	3.774	0.923
3.789	0.895	3.789	0.894

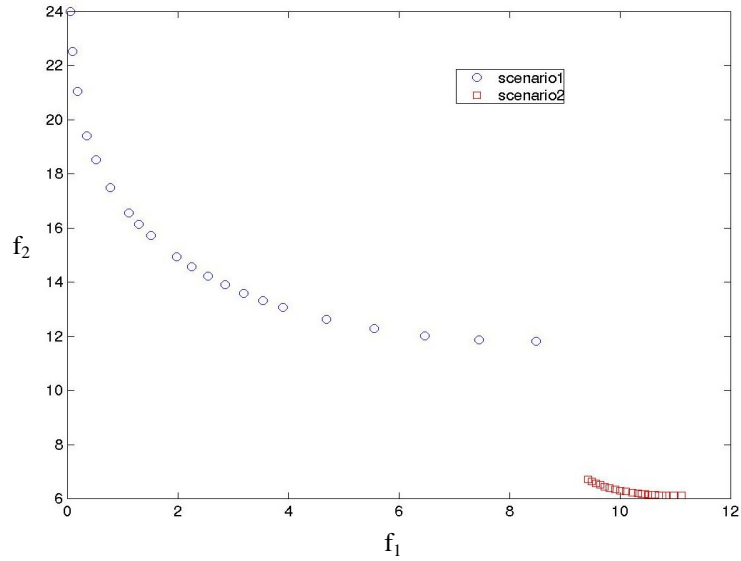


Figure 6. Pareto sets of the IDSPs and the CDSPs of Example 3.1.2

Similar to Example 3.1.1, the Pareto curves have no point in common. Even though the two Pareto curves may be plotted on the same figure, i.e., bi-objective space (f_1, f_2) , their location with respect to each other is meaningless. While this statement is trivial to the expert reader, this concept must be emphasized. The fact the two Pareto curves of Figures 4 and 6 do not intersect does not imply that the two scenarios have no common efficient design. In other words, the Pareto curves in the criteria space do not convey any information regarding common efficient designs. It is only by examination of the sets of efficient solutions plotted in the design space that common efficient designs can be found. In fact, contrary to example 1, example 2 has common efficient designs even though the two Pareto curves do not intersect.

Table 1 reveals that the two scenarios have three common efficient designs. These points are shown in bold in the table and have some negligible differences due to numerical inaccuracies. Based on the numerical representation of the efficient sets, one might think that these points are the only common designs for the two scenarios. A closer look at the design space, shown in Figure 5, helps to see that there are infinitely many common points. In fact, the entire efficient set of scenario 2 appears to be also efficient for scenario 1. It must be recognized that this realization is possible for simple mathematical problems, but may be unfeasible in most large-scale engineering problems. Those additional common designs are not contained in the table produced by the approximation algorithm. Clearly, however, if more iterations of the algorithm were run, then more common points might be found.

3.2. All-at-once approach

The all-at-once approach is traditionally used to solve the bi-scenario bi-criteria problem (3.1) and we do not advocate it. We present it, however, to contrast with the scenario-oriented approach.

We convert the bi-scenario bi-criteria problem into the four-criteria problem in which feasible designs are constrained to the intersection of the feasible sets X^1 and X^2 :

$$\begin{aligned} \min \quad & [f_1^1(x), f_2^1(x), f_1^2(x), f_2^2(x)] \\ \text{s.t.} \quad & x \in X^1 \cap X^2 \end{aligned} \tag{3.5}$$

Efficient designs of problem (3.5) are considered solutions of problem (3.1) and can be found using any suitable method available in the literature (see Section 2.1).

Definition 3.2.1 A design x^* is said be a *solution to the bi-scenario bi-criteria problem* (3.1) if it is efficient for the all-at-once problem (3.5).

Though this approach seems to be naturally straightforward, it has certain drawbacks. The dimension of the criteria space increases, which makes the analysis more difficult for the designer. The greater the dimension of the criteria space, the more difficult the physical and geometrical perception of the problem. In addition, an increase in the dimension of the criteria space makes analyses of tradeoffs between criteria more cumbersome. Additionally, as we showed in Section 2.2, a Pareto solution of a problem with more criterion functions may not remain Pareto when some criteria have been eliminated. Therefore, when solving problem (3.5), one may find solutions that are not Pareto for any of the two CDSPs.

3.3. Scenario-oriented approach

In contrast to the all-at-once approach, in every stage of the scenario-oriented approach applied to a bi-scenario bi-criteria problem the objective space is confined to two dimensions.

Scenario problems are always treated separately which means that only the Pareto and efficient sets of IDSPs and CDSPs are analyzed. This approach is justified on the objective and subjective grounds. The former encompasses existing technical support for designers. If the IDSPs and CDSPs are bi-criteria, as it is in our case, the designers will be equipped with computer graphics tools guaranteeing easy visual access to all sets in the objective space. The latter relates to designers' knowledge and experience. As indicated in the introduction, we believe that it is easier to deal with smaller scenario-oriented problems than with a large overall problem. Designers may have more insight into specifics of an individual scenario, be able to better define subjective preferences and make better tradeoff decisions, and eventually develop a customized design for two (or more) scenarios.

Defining a solution for problem (3.1) is now a challenging task. An ideal solution would be a design $x^* \in X^1 \cap X^2$ that is efficient for each IDSP, i.e., $x^* \in X^1_E \cap X^2_E$. In ideal but extremely unlikely situations this intersection may be equal to a single point that becomes the optimal design x^* of problem (3.1). If the intersection contains several or infinitely many points, one has to employ decision analysis techniques to select a preferred design becoming the optimal design x^* for problem (3.1). If the intersection is empty, we solve the CDSPs and follow with a similar analysis. Furthermore, on top of resulting from a mathematical model, a design should be preferred by the designer in order to account for any requirement not considered in the mathematical formulation. In view of this discussion, we propose the following definition of an optimal design for problem (3.1).

Definition 3.3.1 (i) A design x^* is said be *optimal* to the bi-scenario bi-criteria problem (3.1) if it is feasible for each IDSP (3.3) and (3.4) and

1. Pareto for each IDSP and preferred by the designer, or
2. Pareto for each CDSP of type (3.2) and preferred by the designer, or
3. Pareto for one CDSP, satisfactory for the other CDSP and preferred by the designer, or
4. satisfactory for both CDSPs and preferred by the designer.

(ii) A design is considered *satisfactory* for a scenario if its lack of efficiency with respect to the other scenario is smaller than a predefined threshold value.

Although this definition is quite general, we interpret it as a sequence of four conditions. If a condition i , $i = 1, \dots, 3$, holds then the subsequent conditions are not exercised. If a condition i does not hold then we move to the condition $i+1$.

According to this definition, the final solution design x^* is selected from the set of efficient designs $X^1_{EC} \cup X^2_{EC}$. In most cases, however, the set of Pareto solutions is very large and the selection process is complicated and subjective. Being able to reduce the number of solutions to choose from in a systematic manner is a considerable advantage. Since all designs are efficient for at least one scenario, a design that has a low lack of efficiency in the other scenario is considered a good solution for the overall problem. From this definition, a threshold value can be defined to reduce the number of designs of interest. The threshold value is defined such that all the solutions that have a lack of efficiency smaller than or equal to the threshold value are considered acceptable. All other solutions are rejected on the basis that even though they are efficient for one scenario, they are far from being efficient in the other scenario.

We now present a procedure, called the scenario-oriented procedure, for finding candidate designs to be optimal solutions of (3.1) in the spirit of Definition 3.3.1.

The scenario-oriented procedure

STAGE 1: Separately solve the IDSPs 1 and 2 (problems (3.3) and (3.4)). If the efficient sets of these problems have a nonempty intersection and there exists a solution x^* in this intersection preferred by the designer, select x^* as the solution of the bi-scenario bi-criteria problem (3.1). Otherwise proceed to stage 2.

STAGE 2: Separately solve the CDSPs 1 and 2 of type (3.2). Similar to stage 1, check the intersection of these efficient sets. If the efficient sets of these problems have a nonempty intersection and there exists a solution x^* in this intersection preferred by the designer, select x^* as the solution of the bi-scenario bi-criteria problem (3.1). Otherwise proceed to stage 3.

STAGE 3: Given the efficient sets X_{EC}^s computed in the previous stage, and the sets of outcomes $Z_{EC}^s = \{f^s(x), x \in X_{EC}^s\}$ for $s = 1, 2$, compute the sets of outcomes Z_{ij} , $i, j = 1, 2, i \neq j$, in the respective alternate scenarios defined as $Z_{12} = \{f^1(x), x \in X_{EC}^2\}$ and $Z_{21} = \{f^2(x), x \in X_{EC}^1\}$. Then compute the lack of efficiency, $LOE(x)$, of each efficient design x , $x \in X_{EC}^s$, of a scenario for the respective alternate scenario using the approximation-based approach. For instance, if x_0^1 is an efficient design for scenario 1, its lack of efficiency for scenario 2 is quantified by computing the distance between its outcome in scenario 2 and the Pareto set of scenario 2. Using the lack of efficiency, all designs are compared to a pre-defined threshold value d_{max} . For all $x \in X_{EC}^1 \cup X_{EC}^2$, if $LOE(x) > d_{max}$, reject x . If $LOE(x) \leq d_{max}$, consider x as a design of interest. Then select a solution x^* from the set of designs of interest. The definition of a design of interest ensures that the final solution x^* will be selected from a set of designs that are efficient in one scenario and satisfactory in the other scenario. At this point, we assume that a design preferred by the designer can be found at the conclusion of stage 3. In the case where none of the designs of interest are preferred by the designer, a fourth stage must be defined. This stage is the subject of further research.

It must be recognized that this procedure does not offer a method to select the final solution x^* from a set of designs. Rather, the procedure is partly a tool that reduces the number of designs to choose from. At any stage, it is up to the designer to exercise judgment for the selection of x^* based on experience, preferences, or additional criteria.

Concerning the tradeoff between scenarios, the scenario-oriented procedure allows specifying preferences between scenarios by defining a different threshold value for each scenario. For instance, if scenario 1 is considered more important than scenario 2, the threshold value for scenario 1 would be smaller than that for scenario 2. As a result, more efficient designs of scenario 2 would be rejected and the set of designs of interest would comprise predominantly solutions that are efficient for scenario 1. Also, all designs of interest would be either efficient or with a low lack of efficiency in scenario 1.

4. Examples

Based on the scenario-oriented procedure, a fully automated computer code was developed. The designer must define each scenario optimization problem as well as the pre-defined threshold value d_{max} if stage 3 of the procedure is reached. The selection of d_{max} is aided by considering the range of variation of the lack of efficiency of all designs. This concept is exemplified in this section with numerical applications. We apply the scenario-oriented procedure to a structural three-bar truss problem and a tractor-trailer dynamics problem.

4.1. Three-bar truss problem

The three-bar truss shown in Figure 7 is subject to static loading applied at node D. Two scenarios corresponding to two load cases are considered; i.e., (F_x, F_y) is equal to $(20 \text{ kN}, -20 \text{ kN})$ for scenario 1 and $(-20 \text{ kN}, -20 \text{ kN})$ for scenario 2. The total weight and the displacement at node D are two criteria to be minimized with respect to the three cross-sectional areas, x_i , $i = 1, \dots, 3$, of the three bars. Lower and upper bounds on the cross-sectional areas are defined. In addition, the normal stress is constrained by a maximum allowable stress of 200 MPa.

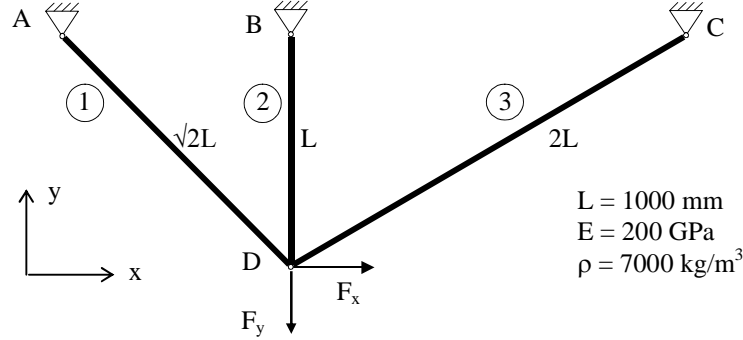


Figure 7. Three-bar truss under static loads

The corresponding IDSPs are written

$$\left\{ \begin{array}{ll} \min & [w(x_1, x_2, x_3), d^s(x_1, x_2, x_3)] \\ \text{w.r.t.} & x_i \quad i = 1, \dots, 3 \\ \text{s.t.} & N_i^s(x_1, x_2, x_3)/x_i \leq 200 \quad i = 1, \dots, 3 \\ & 10 \leq x_i \leq 200 \quad i = 1, \dots, 3 \end{array} \right\} \quad s = 1, 2 \quad (4.1)$$

where w is the total structural weight; d^s is the amplitude of displacement at node D for scenario s ; and N_i^s is the internal force in bar i for scenario s . The formulations of w , d^s , and N_i^s in terms of x_i were explicitly derived based on structural mechanics. The scenario-oriented approach presented in Section 3.3 is now applied as follows.

STAGE 1: We solve the two IDSPs as two bi-criteria optimization problems using the weighted-Tchebycheff method and the SQP Matlab[®] optimizer to produce 22 efficient points for each scenario. The two efficient sets are graphically represented in Figure 8. In an attempt to improve the three-dimensional visualization of the efficient sets, their respective projections on the three coordinate planes $x_1=0$, $x_2=0$, and $x_3=0$ are plotted. Upon analysis of the figure, it appears that the two efficient sets are relatively far apart and share only one common efficient design $(200, 200, 200)$. This solution is efficient for both scenarios, which means that it is the solution of the problem as formally written in (4.1). However, this solution lies on the boundary of the feasible set and yields an extreme Pareto outcome. This means the minimization of one of the two objectives (displacement, in this case) is excessively emphasized at the expense of the other objective (weight). Therefore, from a practical point of view, this design may not be preferred by the designer. Assuming that it is the case, we go to stage 2.

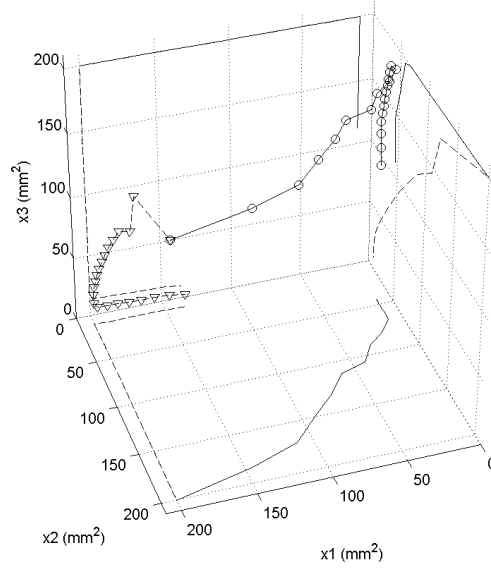


Figure 8. Efficient sets of IDSPs and their projections on coordinate planes, for $s = 1$ (solid and circles) and $s = 2$ (dash and triangles)

STAGE 2: We solve the two CDSPs as two bi-criteria optimization problems. Note that a CDSP includes all the constraints of both IDSPs. The two efficient sets and their projections are graphically represented in Figure 9. Similar to the previous stage, we find that the two efficient sets are far apart and share only two common designs (200, 200, 200) and (95.3, 81.2, 87.0). However, yielding extreme Pareto outcomes, these designs may not be preferred by the designer. We then go to stage 3.

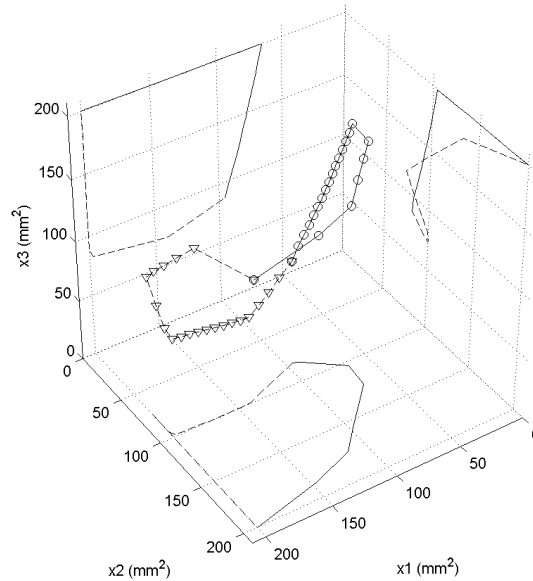


Figure 9. Efficient sets of CDSPs and their projections on coordinate planes, for $s = 1$ (solid and circles) and $s = 2$ (dash and triangles)

STAGE 3: We first compute the sets Z_{21} and Z_{12} . We then apply the approximation-based approach and measure the lack of efficiency of all designs in their respective alternate scenarios, i.e., we calculate the Euclidean distances in the normalized objective space between Z_{E1} and Z_{12} and between Z_{E2} and Z_{21} . The four sets Z_{E1} , Z_{E2} , Z_{12} , and Z_{21} are plotted in Figure 10 and the distances, which represent the lack of efficiency of the designs, are plotted in Figure 11.

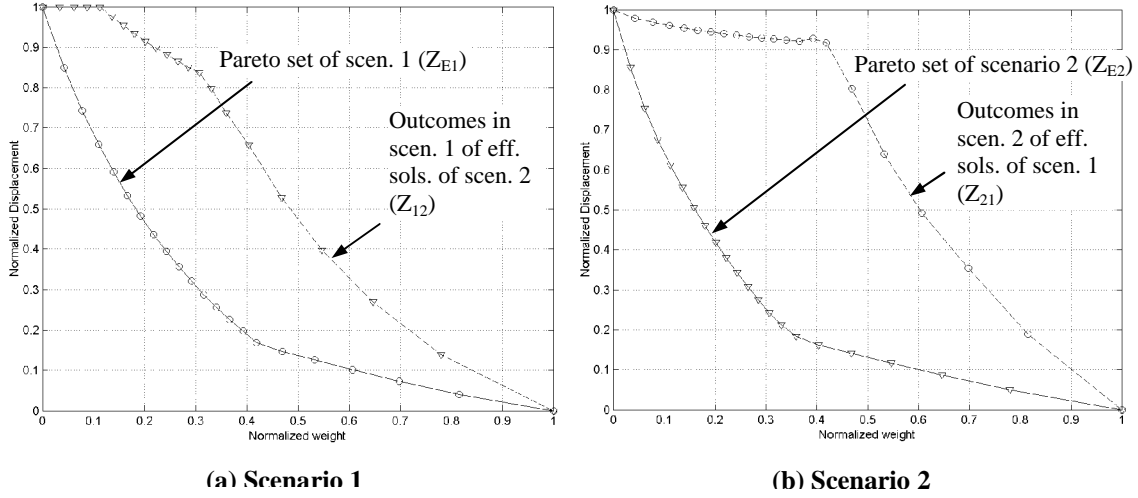


Figure 10. Pareto sets of CDSPs and their outcomes in respective alternate scenarios for $s = 1$ (solid and circles) and $s = 2$ (dash and triangles) after normalization

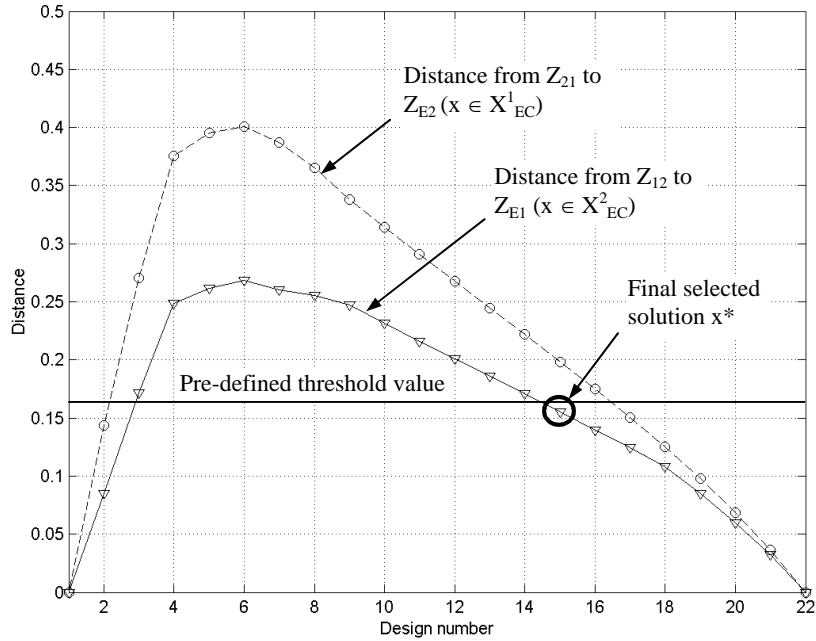


Figure 11. Lack of efficiency in scenario s of efficient designs of the respective alternate scenario after normalization, $s = 1$ (solid and triangles) and $s = 2$ (dash and circles)

Assume that the threshold value is defined as 40 percent of the maximum lack of efficiency, i.e., 0.16, for both scenarios. By rejecting designs that have a lack of efficiency greater than 0.16, the set of acceptable solutions is reduced from 44 to 18 designs.

From this reduced set, an additional consideration can be exercised to select a final solution for the problem. This consideration is based on the fact that extreme Pareto solutions, i.e., one objective is overwhelmingly predominant over the other objective, are generally not preferred. Therefore, a good solution should be as far as possible from extreme Pareto outcomes, which are at both ends of the Pareto sets in the case of the three-bar truss problem. The corresponding final solution x^* is highlighted in Figure 11.

Comparison with all-at-once approach

In order to validate the proposed scenario-oriented approach, the results are compared to those of the all-at-once (AAO) approach. The optimization problem (3.5) is solved using the weighted-Tchebycheff method with 22 values of the weighting coefficient for each criterion function, which results in 2024 efficient designs. These solutions are graphically represented in the design space along with the efficient solutions of both CDSPs (linearly interpolated) in Figure 12. It must be clear that the 2024 points are efficient for the AAO problem (3.5), and the linear interpolations are linear segments between efficient solutions of the CDSPs. In addition, Figure 12 includes the projections of all sets on the three coordinate planes. The reader is encouraged to refer to Figure 9 for direct comparison between the efficient solutions of AAO, CDSP1, and CDSP2.

The outcomes of the efficient solutions of the AAO problem are plotted in the scenario-oriented objective spaces in Figure 13. Refer to Figure 10 for direct comparison with the CDSPs.

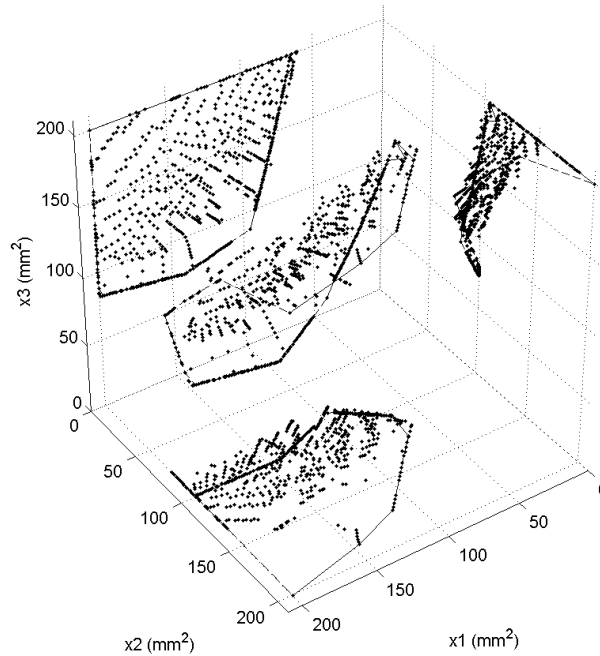


Figure 12. Efficient solutions of the AAO problem (dots), linear interpolations of sets of efficient solutions of CDSPs, $s = 1$ (solid) and $s = 2$ (dash), and all projections on coordinate planes

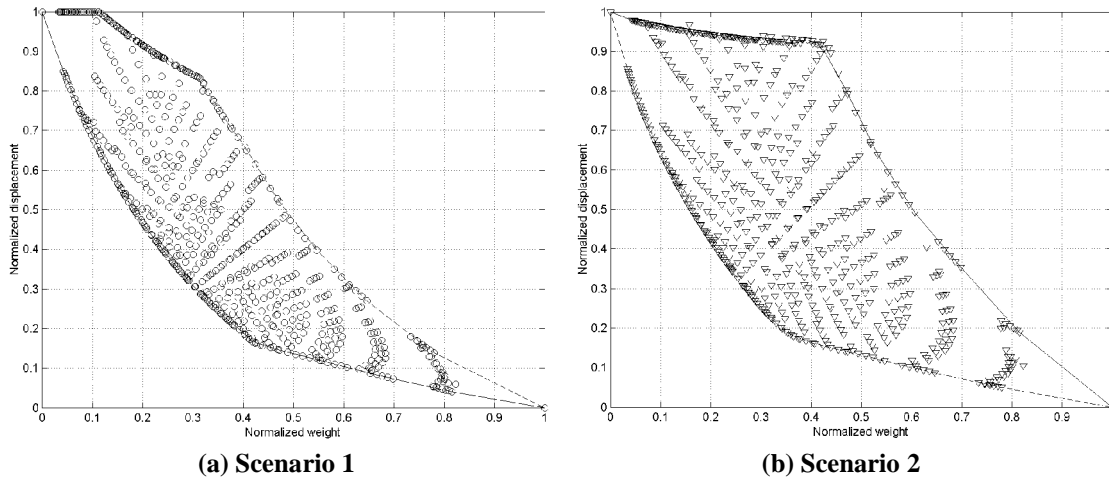


Figure 13. Outcomes of the efficient solutions of the AAO problem in scenario s , $s = 1$ (circles) and $s = 2$ (triangles), and linear interpolations of Pareto sets of CDSPs, $s = 1$ (solid) and $s = 2$ (dash), after normalization

From Figures 12 and 13 it appears, as expected, that the entire efficient set of the AAO problem is distributed on and between the efficient sets of the CDSPs. With a closer look, one can see that many solutions exactly lay on the efficient sets of the CDSPs. In fact, 65% of the AAO solutions have a lack of efficiency lower than 1% of the maximum lack of efficiency for either scenario 1 or scenario 2 (and 55% have a lack of efficiency lower than 0.1%). This means that 65% of the AAO solutions can be considered efficient for at least one scenario. The remaining 35% are designs that are found only by the AAO approach and cannot be captured by the scenario-oriented approach.

The fact that the majority of the AAO efficient designs can be captured by solving the CDSPs is an advantage for the scenario-oriented procedure. However, even though the remaining 35 percent of the AAO solutions are fewer, the designer may be predominantly interested in these solutions since many of them may be satisfactory in all scenarios. This statement is the basis for the future development of a methodology targeting stage 4 of the scenario-oriented procedure.

4.2. Tractor-trailer dynamics problem

The previous engineering example falls in the category of continuous multi-criteria optimization problems and has the advantage of being simple enough for illustration purposes. In addition, it is a pertinent structural design problem scalable to much greater sizes with similar trends and conclusions expected to occur. An exact optimization solver (SQP) was used to solve all optimization problems related to this example.

In this section, we discuss the applicability of the scenario-oriented procedure to a tractor-trailer design problem, which falls in the category of non-continuous optimization problems (discontinuities in design variables, criteria, and/or constraints). To be precise, in the tractor-trailer problem the discontinuities are due to the time-step-dependency of the vehicle dynamics analysis model. Heuristic and simulation-based methods such as genetic algorithms, simulated annealing, and design of experiments (DoE) are generally used to solve this type of problem.

The tractor-trailer, shown in Figure 14, is optimized for two standard maneuvers that correspond to two different scenarios, namely, the single lane change maneuver (SL) and the ramp steer maneuver (RS) [22]. The design variables include 21 physical parameters that have a significant effect on vehicle performance. They include, among others, the tire stiffness, the locations of the centers of gravity, the wheel-base length, and the track widths. For each scenario, two objectives are considered. For the single lane change (scenario 1), the load transfer ratio (LTR) and the rearward amplification factor (RWA) are to be minimized. For the ramp steer maneuver (scenario 2), the understeer coefficient (K_u) and the static rollover threshold (SRT) are to be maximized. These four objective functions are also referred to as performance indices. The numerical model used in this research is the ArcSim tractor-trailer model developed at the Automotive Research Center at the University of Michigan [1].

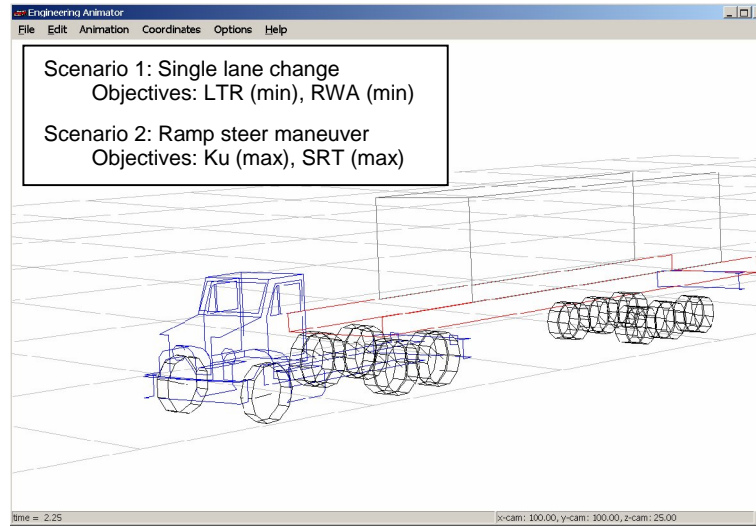


Figure 14. ArcSim tractor-trailer model [1]

Latin Hypercube Design of Experiments

The optimization problem is solved using the Latin hypercube DoE technique with 1000 points [14]. The idea behind the Latin hypercube technique is to span the entire 21-dimensional design space with a well-distributed sampling. For each point and each scenario, a time-dependent numerical simulation is executed, upon which the criterion functions (performance indices: LTR, RWA, K_u , SRT) are computed.

In this particular optimization problem, since no constraints are defined, the IDSPs and CDSPs are equivalent, which means that stages 1 and 2 of the scenario-oriented procedure are identical. The DoE technique is carried out by evaluating the performance indices of the 1000 designs conceived by the Latin Hypercube sampling. The results, shown in Figure 15, are then post-processed to extract the Pareto solutions of the CDSPs 1 and 2. In addition, the Pareto solutions of the AAO problem are plotted to allow further discussion on the benefits of the procedure. One can see that there is no common efficient design between the two scenarios. Therefore, we go to stage 3 by considering the sets of outcomes of the efficient designs of the CDSPs in their respective alternate scenario, i.e., Z_{12} and Z_{21} . We then compute the lack of efficiency of all efficient designs using the approximation-based approach (Figure 16). By

comparison with a pre-defined threshold value, the set of designs of interest can be reduced to a few designs from which a final solution can be selected.

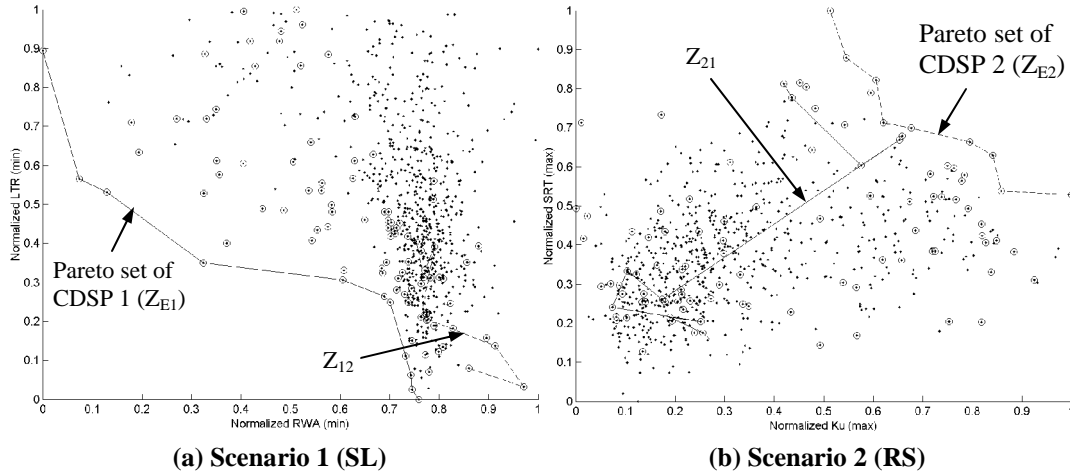


Figure 15. Outcomes of 1000 designs (dots) in both scenarios, Pareto set of the AAO problem (circled dots), Pareto sets of the CDSPs, $s = 1$ (solid) and $s = 2$ (dash) and their images in respective alternate scenarios after normalization

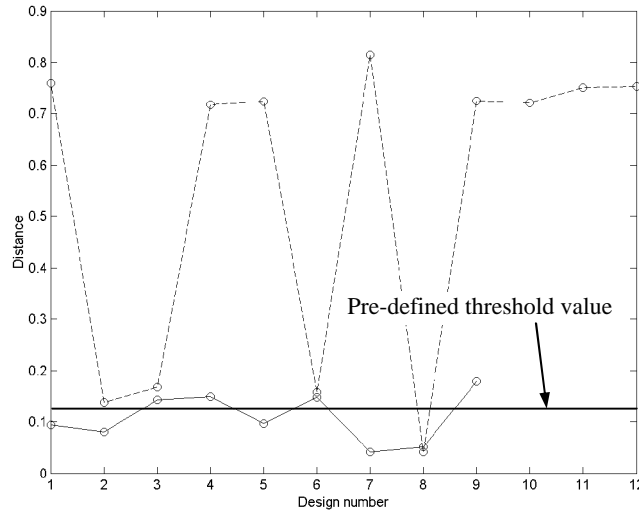


Figure 16. Lack of efficiency in scenario s of efficient designs of the respective alternate scenario after normalization, $s = 1$ (solid) and $s = 2$ (dash)

By using methods such as DoE, there is no significant gain in computational effort with the scenario-oriented approach as opposed to the AAO approach. This is due to the fact that the success in finding Pareto solutions relies on scanning the entire design space on a point-by-point basis without any evolving search during the optimization process and without consideration of a neighborhood around each point in the outcome set. For each point, performance analyses pertaining to each scenario must be executed to find the efficient sets of each CDSP, which requires as much effort as executing the performance analyses to find the efficient set of the

AAO problem. Figure 15 shows that only a few of the Pareto solutions of the AAO problem (circled dots) are captured by the scenario-oriented procedure. Therefore, since all the AAO Pareto solutions can be extracted with the same amount of effort, the scenario-oriented procedure is not recommended. However, the concept of lack of efficiency can be applied to the set of AAO Pareto solutions in order to reduce the set of designs of interest to a minimum, from which a final solution can be selected.

Multi-objective Genetic Algorithm

Evolutionary methods such as multi-objective genetic algorithm (MOGA) are attractive alternatives to DoE often used for this type of problem [15]. Similar to exact methods (e.g., gradient-based), MOGA is limited by the number of criterion functions to consider. In other words, the optimization process becomes increasingly computationally intensive when the number of criterion functions increases, and the chance of success in finding Pareto solutions decreases accordingly. Therefore, the scenario-oriented procedure is expected to be significantly advantageous in this regard.

MOGA is an optimization technique developed to search for an entire Pareto set in a single execution. Therefore, solving the CDSPs of the tractor-trailer dynamics problem using MOGA would lead to Pareto solutions similar to the ones of the CDSPs shown in Figure 15. Following stage 2, one can see that these Pareto sets do not have any common efficient designs. Therefore, we go to stage 3. We then compute the lack of efficiency of all efficient designs and reduce the set of designs of interest by comparison with a pre-defined threshold value.

5. Discussion and Conclusion

The scenario-oriented procedure for multi-scenario multi-criteria optimization problems was presented and exemplified by means of a continuous structural problem and a discrete vehicle dynamics problem. Its advantages and shortcomings were discussed and compared to that of the traditional all-at-once approach. The procedure is based on considerations in both the design and objective spaces, which complement each other. The main purpose of the scenario-oriented approach is to be able to deal with a series of small multi-criteria design problems as opposed to a single large multi-criteria problem. This results into a tradeoff between (1) capturing all the solutions and (2) being able to deal with a large number of scenarios and criteria in terms of computational effort, graphical and cognitive perception of the problem, decision-making, and scenario-oriented customization.

Even though the scenario-oriented procedure was exemplified with bi-scenario bi-criteria problems, there is no limit on the number of scenarios and criteria. In fact, the method becomes even more beneficial as the scale of the problem increases. The definition of the lack of efficiency based on a distance between an outcome and a Pareto curve is independent of the number of criteria. With more than two scenarios, we propose to use, for example, a weighted sum of the lack of efficiency quantities of all scenarios and use this total value to reduce the set of solutions. The coefficients of this weighted sum become then a means to control preferences between scenarios.

The proposed procedure was shown to be beneficial for problems solved using gradient-based methods and evolutionary methods such as MOGA. However, it is not as beneficial when using DoE since the computational efforts involved in the scenario-oriented and the AAO approaches are virtually the same. In this case, the AAO approach has the advantage of finding

all the designs of interest from which a final solution should be selected. We then argue that this selection process can be facilitated by means of the scenario-oriented lack of efficiency presented in this paper.

Finally, the fourth stage of the scenario-oriented procedure is the subject of further research. This stage is dedicated to seeking solutions that are satisfactory for all scenarios without solving the AAO problem. We believe that these solutions are of prime interest to the designer and decision maker.

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